

Here is pretty much all the Intermediate Tier content we could fit onto an A3 sheet of paper, including all the formulae you are required to know for GCSE. An → points to an illustrative example. NM = Numeracy and GCSE Mathematics; M = GCSE Mathematics only. Pin this to a wall, keep it on your desk, carry it in your bag, make notes on it (sorry, don't take it into the examination)...

Percentages NM

Increase or decrease by a percentage.
 → Increase £45 by 8%.
 $\frac{8}{100} \times 45 = 3.6$
 $45 + 3.6 = \pounds 48.60$

Express as a percentage
 → What is a mark of 36 out of 78 as a percentage?
 $\frac{36}{78} \times 100 = 46.2\% \text{ (1dp)}$

Percentage profit and loss NM

Express the profit or loss as a percentage of the original amount.
 → I buy an antique for £250 and sell it for £420. What is my percentage profit?
 $\frac{420-250}{250} \times 100 = 68\%$

Reverse percentage NM

Don't find percentage of new amount
 → The price of a jacket is reduced by 35% to £156. What was the original price of the jacket? [note that £156 is 65% of the original]
 $65\% = \pounds 156$
 $1\% = \pounds 156 \div 65 = \pounds 2.40$
 Original price is $100 \times \pounds 2.40 = \pounds 240$

Compound interest NM

Total accrued = $P \left(1 + \frac{r}{100}\right)^n$
 → I invest £600 at 3% compound interest. What is my account worth after 5 years?
 $\pounds 600 \times \left(1 + \frac{3}{100}\right)^5 = \pounds 695.56$

Error intervals NM

Find the range of numbers that will round to a given value.
 → $x = 5.83$ (2 decimal places)
 $5.825 \leq x < 5.835$
 → $y = 46$ (2 significant figures)
 $45.5 \leq y < 46.5$
 Note use of \leq and $<$, and that the last significant figure of each is 5

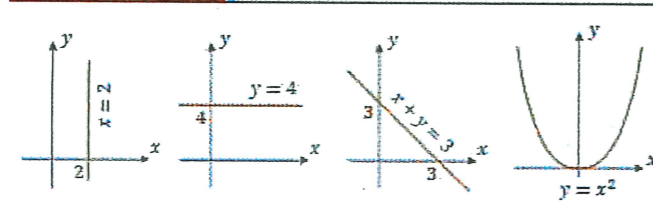
Powers and roots NM

Special indices: for any value a :
 $a^0 = 1$
 $a^{-n} = \frac{1}{a^n}$
 $a^{\frac{1}{n}} = \sqrt[n]{a}$
 $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$
 $3^{\frac{1}{3}} = \sqrt[3]{3} = 2$

Standard form NM

Numbers of the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer.

Standard graphs M



Laws of indices NM

For any value a :
 $a^x \times a^y = a^{x+y}$
 $\frac{a^x}{a^y} = a^{x-y}$
 $(a^x)^y = a^{xy}$
 $\frac{a^x}{a^y} = a^{x-y}$
 $\frac{x^a}{x^b} = x^{a-b}$
 $y = mx + c$

Equation of straight line M

$y = mx + c$
 m is the gradient; c is the y intercept.
 → Find the equation of the line that joins $(0, 3)$ to $(2, 11)$
 Find its gradient...
 $\frac{11-3}{2-0} = \frac{8}{2} = 4$
 ...and its y intercept...
 Passes through $(0, 3)$, so $c = 3$
 Equation is $y = 4x + 3$

Parallel, perpendicular lines M

Parallel lines: gradients are equal; perpendicular lines: gradients are negative reciprocals.
 → $y = 2x + 3$ and $y = 2x - 5$ are parallel to each other; $y = 2x + 3$ and $y = -\frac{1}{2}x + 3$ are perpendicular

Sequences NM

Triangular numbers:

| 1st | 2nd | 3rd | 4th | 5th |
|-----|-----|-----|-----|-----|
| 1 | 3 | 6 | 10 | 15 |

Square numbers ($n^2 = n \times n$):

| | | | | |
|----------------|----------------|----------------|----------------|----------------|
| 1 ² | 2 ² | 3 ² | 4 ² | 5 ² |
| 1 | 4 | 9 | 16 | 25 |

Cube numbers ($n^3 = n \times n \times n$):

| | | | | |
|----------------|----------------|----------------|----------------|----------------|
| 1 ³ | 2 ³ | 3 ³ | 4 ³ | 5 ³ |
| 1 | 8 | 27 | 64 | 125 |

n th term of an arithmetic (linear) sequence is $an + d$

→ n th term of 5, 8, 11, 14, ... is $3n + 2$ (always increases by 3, first term is $3 \times 1 + 2 = 5$)

Brackets M

Expand; multiply out brackets...
 $(x + a)(x + b) = x^2 + ax + bx + ab$
 → $(2x - 3)(x + 5) = 2x^2 - 3x + 10x - 15 = 2x^2 + 7x - 15$

Factorise; put into brackets...

→ Factorise fully $10x^2 + 8xy^2 = 2x(5x^2 + 4y^2)$
 → Factorise $x^2 - 3x - 28 = (x - 7)(x + 4)$

Simultaneous equations M

→ Solve $\begin{cases} 2x + 3y = 11 \\ 3x - 5y = 7 \end{cases}$
 Multiply to match a term in x or y
 $\begin{cases} 10x + 15y = 55 \\ 9x - 15y = 21 \end{cases}$
 Add or subtract to cancel...
 $19x = 76$, so $x = 4$
 Finally, substitute and solve...
 $2 \times 4 + 3y = 11$, so $y = 1$

Trial and improvement M

→ Solve $x^2 + 2x = 250$ to 1dp, given that $6 < x < 7$
 Trial a value (say 6.5) with $6 < x < 7$
 $6.5^2 + 2 \times 6.5 = 287.6$... (too low)
 Find two consecutive values...
 $6.1^2 + 2 \times 6.1 = 239.2$... (too low)
 $6.2^2 + 2 \times 6.2 = 250.7$... (too high)
 Test intermediate value...
 $6.15^2 + 2 \times 6.15 = 244.9$... (too low)
 Hence $6.15 < x < 6.2$
 ...so solution is closer to 6.2 than 6.1
 $x = 6.2$ (to 1dp)

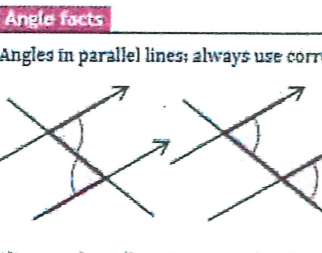
Quadratics M

Solve a quadratic by factorising.
 → Solve $x^2 + 8x + 15 = 0$
 Put into brackets (taking care with negative numbers)...
 $(x + 5)(x + 3) = 0$
 ...then either $x + 5 = 0$ or $x + 3 = 0$
 so that $x = -5$ or $x = -3$

Rearrange a formula M

The subject of a formula is the term on its own. Use rules that "balance" the formula to change its subject.
 → Make x the subject of $2x + 3y = z$
 Here, subtract $3y$ from both sides...
 $2x = z - 3y$
 ...then divide both sides by 2
 $x = \frac{z - 3y}{2}$

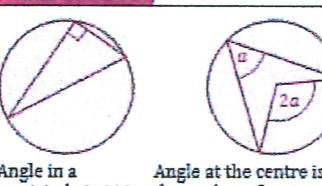
Angle facts M



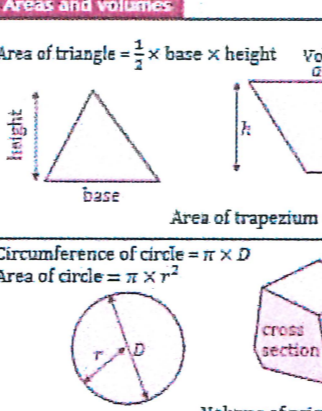
Right angled triangles M

Pythagoras Theorem. Links all three sides.
 $a^2 + b^2 = c^2$
 No angles.
 Trigonometry. Links two sides and one angle.
 SOH | CAH | TOA
 $\sin \theta = \frac{\text{OPP}}{\text{hyp}}$ $\cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\tan \theta = \frac{\text{OPP}}{\text{adj}}$
 Use "2ndF" or "SHIFT" key to find a missing angle
 The longest side of any right angled triangle is the hypotenuse; check that your answer is consistent with this.

Circle theorems M



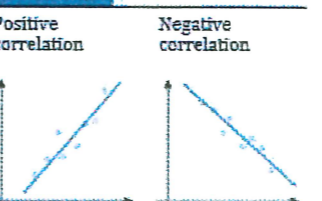
Areas and volumes M



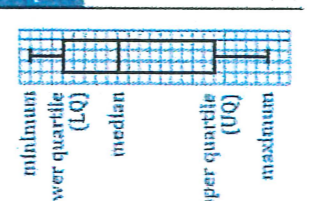
Transformations M

- Reflection: Line of reflection, Vector
- Rotation: Centre of rotation, Angle of rotation, Clockwise or anticlockwise
- Enlargement: Centre of enlargement, Scale factor (if SF < 1 the shape will get smaller).

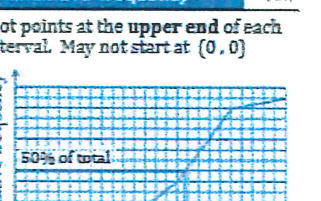
Correlation NM



Box plots NM



Cumulative frequency NM



Measures of spread NM

Range = maximum - minimum
 Interquartile range (IQR) = UQ - LQ

Averages NM

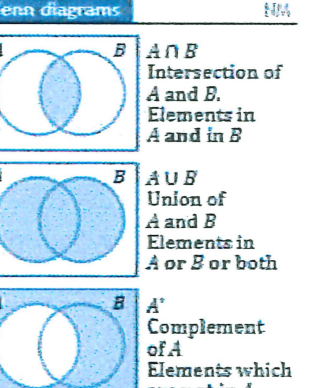
Mode: most frequently occurring
 Median: put the data in numerical order, then choose the middle one
 Mean = $\frac{\text{total of items of data}}{\text{number of items of data}}$

Tabulated data

| x | f | f × x |
|-------|----|--------------|
| 7 | 17 | 7 × 17 = 119 |
| 8 | 9 | 8 × 9 = 72 |
| 9 | 4 | 9 × 4 = 36 |
| Total | 30 | 227 |

Mean = $227 \div 30 = 7.57$ (2dp)
 Mode = 7 [has the highest frequency]
 [Note: if data is grouped, eg $5 < x \leq 10$, etc, use the mid interval values]

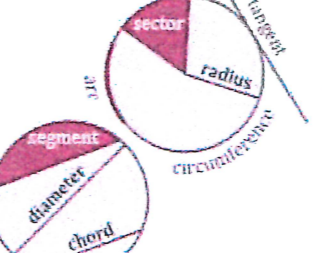
Venn diagrams NM



Probability rules M

Multiply for independent events
 $P(A \text{ and } B) = P(A) \times P(B)$ [AND rule]
 → P(6 on dice and H on coin)
 $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$
 Add for mutually exclusive events
 $P(A \text{ or } B) = P(A) + P(B)$ [OR rule]
 → P(5 or 6 on dice)
 $\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$
 Apply these rules to tree diagrams

Parts of a circle NM



Metric - imperial conversions NM

8 kilometres ≈ 5 miles
 1 kilogram ≈ 2.2 pounds
 1 litre ≈ 1.75 pints
 → I am driving at 35mph. The speed limit is 50kph. Am I breaking the speed limit?
 $35 \div 5 = 7$
 $7 \times 8 = 56$ kilometres
 Yes I am breaking the speed limit

Use triangles for the interior angles of any polygon...

